Field-induced transition from parallel to perpendicular parametric pumping for a microstrip transducer

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Microstrip transducers used for the excitation of spin waves in magnetic films possess two characteristic properties: high spatial localization of the microwave magnetic field and the presence of field components parallel and perpendicular to the bias field. Here, the effects of these features on the process of parametric pumping are presented. By microwave measurements of the spin-wave instability threshold a transition from parallel pumping to perpendicular pumping at the critical field H_c with the minimal threshold is observed. This transition is accompanied by a sharp threshold increase above the critical field due to the spatial confinement of the pump region.

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Microstrip and coplanar transducers have become a standard instrument in magnetism. For the forced excitation of spin waves they posses several advantages. They can be applied to thin films and nano-scaled magnetic structures. When used to realize parametric pumping, the pump region is accessible with optical techniques and the applied microwave power is concentrated into a very high pump field.

Parametric pumping by itself plays an important role in experiments on fundamental properties of magnetic excitations as well as in applications since it allows the amplification, shaping and processing of microwave signals [1, 2, 3]. One of the most exciting applications of this technique was the recent observation of Bose-Einstein condensation of magnons at room temperature [4].

The original theory on parametric pumping was developed by Suhl [5] and Schlömann [6] who considered microwave pump fields oriented perpendicular and parallel to the bias magnetic field, respectively. In the first case, known as subsidiary absorption, the pump field excites a uniform precession which couples to spin-wave modes. For parallel pumping, the pump field directly amplifies spin-wave modes. As for subsidiary absorption this is a threshold processes which only sets in above a certain threshold pump field. The theoretical predictions for the threshold value were successfully tested using cavity resonators which create a spatially uniform pump field [7, 8, 9].

The pump field created by microstrip and coplanar transducers, however, is strongly localized and non-uniform. This affects the pump process not only in the beneficial ways mentioned above. It is, for instance, well known that the localization plays an important role in the amplification of traveling spin waves [10, 11]. But, so far the general question how the use of a microstrip transducer affects the pump process, remains unanswered. In particular, the magnetic pump field around the trans-

To answer the question if this assumption is feasible, the threshold of parametric instability for a microstrip transducer is investigated. It is shown that below the critical field H_c which corresponds to the minimal threshold power and is given by $H_{\rm c} = -2\pi M_{\rm S} +$ $\sqrt{(2\pi M_{\rm S})^2 + (\omega_{\rm p}/(2\gamma))^2}$, where $M_{\rm S}$ is the saturation magnetization, $\omega_{\rm p}$ is the pump frequency and γ is the gyromagnetic ratio, pure parallel pumping is achieved. In this case, the amplified spin waves propagate along the microstrip transducer and are not affected by the spatial localization of the pump field. Above the critical field, the spatial confinement of the pump region leads to an abrupt increase of the threshold because the amplified spin waves leave the pump region. This increased threshold is accompanied by a transition from parallel to perpendicular pumping. At bias magnetic fields $H > 1.2 H_c$, the parametric process at the threshold is completely determined by the perpendicular component of the magnetic pump field.

We observed the influence of the perpendicular field component already at microwave pump powers around

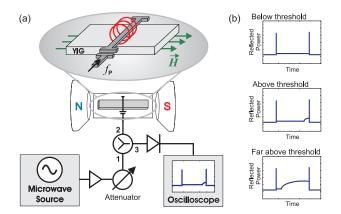


FIG. 1: (Color online) (a) Sketch of the experimental setup. (b) Schematic oscilloscope pictures for different pump powers

ducer has a component oriented perpendicular to the bias magnetic field which is usually neglected in experiments.

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10 mW. The efficiency of the amplification by the perpendicular field component is noteworthy if one takes into account that the frequency of the forced spin oscillations, which serve as a pump source for the parametrically driven spin waves, coincides with the frequency of the external microwave signal and lies in our case around 1500 times the resonance linewidth out of resonance. This has to be kept in mind for numerous experiments with microstrip transducer, where perpendicular pumping could unintentionally occur.

Fig. 1(a) shows the experimental setup. A 20 μ s long microwave pulse with a carrier frequency $\omega_{\rm p}/(2\pi)=14.14$ GHz was generated every 1 ms. Its power was adjusted by a power amplifier and an attenuator. The signal was sent to a half-wavelength microstrip resonator placed on a tangentially magnetized yttrium iron garnet film of 7.8 μ m thickness grown in (111)-direction. A Y-circulator passed the reflected signal to a detector.

The reflected signal depends significantly on the applied pump power (Fig. 1(b)). Below the threshold, almost no signal is reflected due to the good adjustment of the pump resonator. As soon as the applied pump power exceeds the threshold power a kink at the end of the reflected signal appears (middle panel of Fig. 1(b)). This is a consequence of the change in the quality factor and the dynamic detuning of the resonator caused by the excitation of spin waves [12]. In the experiment, the threshold pump power was determined when the kink appeared after 20 μ s at the end of the pump pulse.

The measured data on the threshold pump power is shown in Fig. 2 (a). In general, the data matches previously reported results with cavities: (i) When the pump frequency coincides with twice the frequency of ferromagnetic resonance the threshold power is minimal. The corresponding bias field is defined as H_c . (ii) For fields below H_c the threshold power decreases with increasing applied bias field. (iii) For fields above H_c the threshold power increases with increasing applied bias field. It grows towards infinity as the bias field approaches $H_0 = \omega_p/(2\gamma)$ and half the pumping frequency leaves the spin-wave band (see inset in Fig. 2(a)).

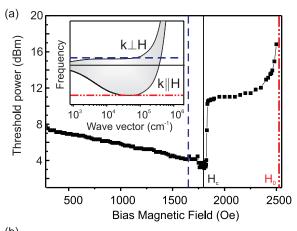
The specialty of the data presented here is a sharp jump which occurs just above H_c . Within a change of the bias field of 15 Oe the threshold increases more than 4 times (7 dB) before it quickly levels off.

To interpret the observed data consider

$$h_{\rm thr} = \min_{k} \frac{\Gamma_k}{V_k} \tag{1}$$

describing the threshold field h_{thr} which depends on the ratio of the spin-wave relaxation frequency Γ_k and the coupling constant V_k of the pump field to the spin wave with wave vector k [13].

At the threshold, only one group of spin waves, which satisfies $\omega_k = \omega_p/2$ and simultaneously has the minimal ratio Γ_k/V_k , is amplified. The decrease of the threshold for low fields is caused by a decrease in the respective spin-wave relaxation frequency Γ_k [6]. Small jumps in



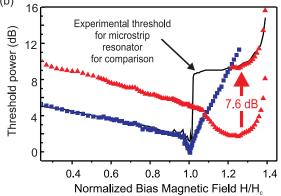


FIG. 2: (Color online) Threshold pump power for (a) microstrip resonator and (b) dielectric resonator in parallel (■) and perpendicular (▲) alignment. The inset in (a) shows a sketched spin-wave spectrum. The horizontal lines in it correspond to the respective vertical lines in the main panel and indicate the relative spectral position of the pumped spin waves.

the threshold are due to the finite film thickness which results in a discrete character of the spectrum [14, 15].

Below H_c the spin waves with the lowest threshold propagate at an angle $\varphi_k = 90^{\circ}$ with respect to the bias field because their coupling $V_k^{\parallel} \sim \sin^2 \varphi_k$ to the parallel pump field is at a maximum. Such spin waves with frequency $\omega_k = \omega_p/2$ do not exist for bias fields above $H_{\rm c}$. Hence, above $H_{\rm c}$ the angle φ_k of the amplified spin waves is smaller than 90°, the coupling V_k^{\parallel} is weaker and the threshold, consequently, higher. In particular, these spin waves possess a non-zero group velocity component parallel to the bias field. Therefore, they leave the area of amplification after a finite time which leads to an increased damping Γ_k . The smaller the pump region, the more striking this effect becomes as has been shown in [16] with dielectric resonators of different sizes. In the investigated case of a microstrip resonator, this effect is expected to be especially pronounced.

The observed leveling of the threshold is directly related to the influence of the perpendicular pump field component. For the perpendicular component the coupling $V_k^{\perp} \sim \sin(2\varphi_k)$ to spin waves of angle $\varphi_k \approx 45^{\circ}$ is

much more efficient than for the parallel field component. For this reason, the perpendicular pump field component starts to govern the processes at the threshold above H_c .

As the applied bias magnetic field approaches H_0 both coupling constants V_k^{\parallel} and V_k^{\perp} decrease leading to a steady increase in the threshold power.

The interpretation was tested by comparing the threshold observed for the microstrip resonator with the threshold of parametric instability for a dielectric resonator.

For this purpose the YIG-sample was placed inside a $4.2 \times 3.1 \times 1.5 \text{ mm}^3$ dielectric resonator. Resonator and sample were then fixed inside a waveguide which was on one end shortened with an adjustable piston and on the other end connected to the setup shown in Fig. 1. Measurements were performed with the same duration of the pump pulse once with the resonator field pointing parallel and once perpendicular to the bias magnetic field which realizes in good approximation pure parallel and pure perpendicular pumping.

The measurement results are shown in Fig. 2(b). The obtained curves agree well with previously reported experiments and the established theory (see references above). Two observations are essential:

- (i) The threshold for perpendicular pumping falls below the one for parallel pumping at a field $H \leq 1.1~H_{\rm c}$. This illustrates, that the perpendicular coupling coefficient V_k^{\perp} surpasses the parallel one V_k^{\parallel} in this field range.
- (ii) The slope of the threshold curve for parallel alignment of the dielectric resonator agrees with the one measured with the microstrip resonator for fields below $H_{\rm c}$. At the same time, the threshold for the perpendicularly aligned dielectric resonator can be brought to coincidence with the microstrip resonator threshold for fields above 1.2 $H_{\rm c}$.

The relative displacement by 7.6 dB of the threshold curves for parallel and perpendicular pumping, which is necessary to match the microstrip resonator threshold curve for low and high bias fields, is caused by the aforementioned spin-wave mediated energy flux from the pump region.

To qualitatively verify the influence of the energy outflow from the pump region on the threshold power, consider the following estimate: Let k be the wave vector of the excited spin wave group with the lowest threshold. For a localized, homogeneous pump region of length l in the direction of the externally applied field, the outflow of spin waves from this region and the relaxation both lead to a dissipation of the spin-wave energy E_k . Hence,

$$\frac{d}{dt}E_k = -2\Gamma_k \cdot E_k - v_k^{\perp} \cdot \frac{dE_k}{dl} = -2\left(\Gamma_k + \frac{v_k^{\perp}}{2l}\right) \cdot E_k \quad (2)$$

where, v_k^{\perp} denotes the component of the spin-wave group velocity perpendicular to the microstrip resonator. According to Eq. (2) the limited size of the pump region leads to an increase in damping by $v_k^{\perp}/2l$ which should be taken into account in Eq. (1) for the threshold field. A rigorous deduction [17] modifies this result slightly and

includes a geometric factor $\pi < b < 2\pi$ which depends on the ratio $\Gamma_k l/v_k^{\perp}$:

$$h_{\rm thr} = \frac{\sqrt{\Gamma_k^2 + (b \ v_k^{\perp}/2l)^2}}{V_k}.$$
 (3)

By looking at the ration of threshold fields $h_{\rm thr}^{\rm ms}/h_{\rm thr}^{\rm dr} \sim \sqrt{P_{\rm thr}^{\rm ms}/P_{\rm thr}^{\rm dr}}$ for microstrip and dielectric resonator, the effective size of the microstrip resonator pump region $l^{\rm ms}$ can be estimated when the bias field is above 1.2 $H_{\rm c}$ and perpendicular pumping takes place.

For $H=2350~{\rm Oe}=1.3~H_{\rm c}$, a typical wave number for parametrically excited spin waves is of the order of $6\times 10^4~{\rm cm}^{-1}$ [15] with a propagation angle $\varphi=25^{\circ}$. This leads to $v^{\perp}\approx 0.07{\rm cm}/\mu{\rm s}$. With an assumed spin-wave damping $\Gamma_k\approx 5\times 10^6~{\rm s}^{-1}$ [10] and the ratio of threshold powers from the experiment $P_{\rm thr}^{\rm ms}/P_{\rm thr}^{\rm dr}=10^{7.6/10}$ one calculates $l^{\rm ms}\approx 100~\mu{\rm m}$. Though this is only a rough estimate it is comparable with the width of the microstrip resonator of 50 $\mu{\rm m}$ and seems, therefore, reasonable.

The calculation confirms, that the increased threshold is caused by the energy outflow from the strongly localized pump region. In general, for higher magnetic field the propagation angle φ of the parametrically excited spin wave group will decrease, however, its wave number k and the group velocity component v^{\perp} will remain almost unchanged. This explains why the observed upward shift of the threshold curve is in fact constant over a whole range of fields from 1.2 H_c to 1.4 H_c .

The above interpretation builds on the perpendicular field component. The threshold curve observed for the parametric generation of spin waves by a microstrip antenna cannot be explained solely by considering the field component parallel to the bias field. This becomes particularly clear in Fig. 2(b) at fields above $1.2\ H_c$. Here, the threshold curve for the parallel aligned dielectric resonator surpasses the one for the microstrip resonator. This is impossible to explain on the base of Eq. (3).

Moreover, the presented explanation for fields above $1.2\ H_c$ solely relies on the perpendicular field component and neglects the presence of the parallel one altogether. Note that, though it exists, it is localized at a different position in space and should only have a minor influence on the threshold power, if any. Overall, the threshold for parametric spin-wave generation by a microstrip transducer can, therefore, be effectively described by using the model of pure parallel pumping for fields below the critical field H_c and the model of pure perpendicular pumping for fields above $1.2\ H_c$.

In conclusion, we have investigated the threshold of parametric spin-wave instability for a microstrip resonator. The experiment shows a characteristic jump in the threshold curve just above the critical bias field. The results are consistently explained as an increase in damping due to the outflow of energy from the strongly localized pump region around the microstrip resonator and the influence of the perpendicular pump field component.

It is possible to change the character of the pumping by a change of the bias field H from below H_c , where the microstrip resonator acts as a source of parallel pumping, to H above 1.2 H_c , where perpendicular pumping (subsidiary absorption) takes place. The comparatively high efficiency of the spin-wave excitation by perpendicular pumping even when the pump frequency is far above

the frequency of ferromagnetic resonance should be considered in any experiment using microstrip transducers.

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